Nonlinear Dyn (2020) 99:1659–1674
https://doi.org/10.1007/s11071-019-05382-y

ORIGINAL PAPER

Robust control for cooperative driving system of heterogeneous vehicles with parameter uncertainties and communication constraints in the vicinity of traffic signals

Dong Chen · Dihua Sun · Hui Liu · Min Zhao · Yang Li · Peng Wan

Received: 19 December 2018 / Accepted: 17 November 2019 / Published online: 27 November 2019
© Springer Nature B.V. 2019

Abstract Taking heterogeneity, internal parameter uncertainties and communication constraints into account, the problem of cooperative driving system of heterogeneous vehicles is studied in the vicinity of traffic signals. This work studies the dynamics of heterogeneous vehicles and response performance from physical perspective and meanwhile analyzes the topology structure and communication constraints between vehicles and roadside equipment from cyber perspective. From the perspective of transportation cyber physical systems, cooperative driving model of heterogeneous vehicles with parameter uncertainties and communication constraints is constructed in the vicinity of traffic signals. We analyze robust stability by adopting Lyapunov–Krasovskii stability theory. According to the limitations of heterogeneity, dynamic uncertainties, communication delays and packet loss, the robust control strategies are proposed by using LMI (linear matrix inequality) method. Through theoretical analysis and numerical simulation, the validity and feasibility of research results are verified, which provides the guidance of control strategies for suppressing traffic congestion.

Keywords Robust control · Cooperative driving system · Heterogeneity · Communication constraints · Transportation cyber physical systems

D. Chen · D. Sun (✉) · M. Zhao · Y. Li · P. Wan
Key Laboratory of Dependable Service Computing in Cyber Physical Society of Ministry of Education, Chongqing University, Chongqing 400044, People's Republic of China
e-mail: d3sun@163.com

D. Chen
College of management science and engineering, Guangxi University of Finance and Economics, Nanning, Guangxi 530003, People's Republic of China

H. Liu
College of Mechanical and Electrical Engineering, Chongqing University of Arts and Sciences, Chongqing 402160, People's Republic of China

D. Chen · D. Sun · M. Zhao · Y. Li · P. Wan
College of Automation, Chongqing University, Chongqing 400044, People's Republic of China

1 Introduction

The emerging information and communication technologies (ICT) facilitate the development of cooperative driving system (CDS), where the vehicles communicate with each other through Vehicle-to-Vehicle communication (V2V) and with the infrastructures through Vehicle-to-Infrastructure communication (V2I). V2V and V2I are together referred to as V2X (Vehicle to X) [1–3]. The feasibility and potential of cooperative driving system have been confirmed by demonstrative experiments and application practices, which aims at the compatibility of traffic safety, efficiency, green and comfort driving [4,5].

Cooperative driving system, an importation application of transportation cyber physical systems (T-CPS), is to guarantee that the vehicles maintain the desired

المنارة للاستشارات
www.manaraa.c

speed and desired space headway (or gap) according to vehicles state information and traffic environment information [6–10].

Some scholars started from the stability of cooperative driving systems and studied platoon control. Both inertia coefficient and time-delay coefficient of vehicles were introduced into three-order typical car-following models, which revealed that vehicle dynamics has strong effect on traffic dynamics and the stability of traffic flow [11]. Taking the velocity difference between the current velocity and the historical velocity into account, Li et al. [12] investigated self-stabilizing effect of vehicles. The stability of vehicle platoon with reaction time of drivers was analyzed, and corresponding feedback control schemes were proposed [13,14]. Al-Jhayyish and Schmidt [15] focused on string stability of heterogeneous vehicle strings with different dynamic properties and considered different possible feedforward strategies.

In addition, the platoon control is more challenging and interesting when the uncertain dynamics and communication delays are considered simultaneously in practical driving conditions. Gao et al. [16] proposed an H-infinity control method for a platoon of heterogeneous vehicles with uncertain vehicle dynamics and uniform communication delay. Zheng et al. [17] used graph theory to establish a unified model to describe the collective behavior of a homogeneous collaborative driving system with external disturbances. The robustness analysis and distributed H_∞ controller synthesis of connected vehicles with undirected topologies were investigated. A dynamic hybrid model of vehicle longitudinal motion was studied when acceleration disturbances, wind resistance, parameter uncertainties and communication uncertainties were considered by Guo and Yue [18]. Zheng et al. [19] proposed a distributed coupled H_∞ robust control method for multi-vehicle systems with identical dynamic controllers and rigid geometric formations. Robust stability, string stability and distance tracking performance of the proposed platoon were discussed. Robust acceleration tracking control of vehicle longitudinal dynamics was proposed by Li et al. [20,21]. They designed several robust controllers for multiple model sets and verified the enhanced robustness of the switching control method. Liu et al. [22] analyzed the confluence control strategy of multiple vehicle queues when the vehicle fleet was subjected to various disturbances. To the authors' knowledge, the problems of cooperative driving system with combined effect of disturbances, vehicle dynamics and communication delays have not been fully solved and still remain challenging and interesting.

The vicinity of traffic signals is the bottleneck of urban traffic which is a relatively small part of urban roads, but it accounts for a large proportion of traffic accidents [23–26]. Because the vicinity of traffic signals can provide the functions such as gathering, steering and evacuation for traffic flows, the vehicles drive and stop periodically in this area with the change of traffic signals. It is easy to decline the performance of cooperative driving, reduce traffic efficiency, increase energy consumption and even lead to traffic accidents (such as rear-end collision) due to internal uncertainties, external perturbations, delays and other factors [27–32].

The control of cooperative driving system in the vicinity of traffic signals will be more challenging when both uncertain dynamics and communication delays are considered. As far as we know, few studies have systematically considered heterogeneity, dynamic uncertainties and the effect of communication constraints for cooperative driving system in the vicinity of traffic signals. The aim of this paper is to investigate cooperative driving of heterogeneous vehicles both considering parameter uncertainties and communication constraints. Cooperative driving model for heterogeneous vehicles is proposed. Using Lyapunov–Krasovskii theory, robust stability of cooperative driving model is analyzed. We further present sufficient conditions for the robust H_∞ controller to ensure robust stability. Considering uncertain external disturbances, the distributed robust control strategies are designed based on linear matrix inequalities (LMIs).

The rest of this paper is structured as follows. In Sect. 2, cooperative driving model for heterogeneous vehicles is established by considering parameter uncertainties and the effect of communication constraints. In Sect. 3, Lyapunov–Krasovskii functional stability theory is used to analyze robust stability of the proposed model. Meanwhile, considering uncertain external disturbances, the distributed robust control strategies are proposed. In Sect. 4, the validity and feasibility of the proposed robust H_∞ control strategies are verified by simulation experiments. The conclusions are summarized in Sect. 5.

المنارة للاستشارات
www.manaraa.com

2 Problem description

2.1 Cooperative driving model

In this section, we establish cooperative driving model in the vicinity of traffic signals and analyze the distributed control strategy. All vehicles must not overtake or change lanes. For cooperative vehicles, some factors are considered, such as air resistance, rolling resistance, communication delays and disturbance inputs. These factors affect dynamic behavior of the vehicles. Some reasonable assumptions are set: (1) The wheels do not slip; (2) the suspension and tire elasticity are negligible; (3) the influence of yaw movement is not considered; (4) the acceleration, deceleration, brake and cruising of the vehicles are determined by the control inputs.

In order to reveal many important features of the practical vehicle dynamics, such as inertia delay of transmission system, the scholars proposed the third-order dynamic system. The longitudinal dynamic model of cooperative driving is generally nonlinear, which includes power transmission system, rolling resistance, air resistance, gravity, etc. The model is expressed as follows:

$$\dot{p}_n(t) = v_n(t) \tag{1}$$

$$\begin{aligned}\frac{\eta_n}{r_n} T_n(t) &= m_n \dot{v}_n(t) + C_{A,n} v_n^2(t)\\ &\quad + m_n g C_r \cos(\theta(p(t))) + m_n g \sin(\theta(p(t)))\end{aligned} \tag{2}$$

$$\varsigma_n \dot{T}_n(t) + T_n(t) = T_{\mathrm{des},n}(t) \tag{3}$$

where $p_n(t)$ and $v_n(t), n = 1, 2, \ldots, N$ are the position and speed of the n-th vehicle at time t, respectively. Moreover, the position and speed of head car are set as $p_0(t)$ and $v_0(t)$. m_n is the vehicle mass. $C_{a,n}$ is the coefficient of air resistance, C_r is the coefficient of rolling resistance, g is the acceleration coefficient of gravity, $T_n(t)$ is the driving or braking force, $T_{\mathrm{des},n}(t)$ is the desired driving or braking force, ς_n is the inertial delay, r_n is the wheel radius, and η_n is the transmission efficiency of power transmission system. $\theta(p)$ is the road slope angle. Considering the horizontal urban road environment, the slope angle is close to 0. So we assume $\cos(\theta(p)) \approx 1$, $\sin(\theta(p)) \approx \theta(p) \approx 0$.

The linear feedback technique is used to transform the nonlinear model into linear model for feedback controller design as follows:

$$\begin{aligned}T_{\mathrm{des},n}(t) &= \frac{1}{\eta_n T_n(t)}[C_{A,n} v_n(t)(2\tau_n \dot{v}_n(t) + v_n(t))\\ &\quad + m_n g c_r + m_n g \theta(p) + m_n u_n(t)] r_n\end{aligned} \tag{4}$$

where $u_n(t)$ is the control input, which represents the desired acceleration.

Equation (4) plays a key role in the process of vehicle feedback linearization control: (1) The linearization of the dynamics equation of the nth vehicle is realized; (2) the model is simplified by removing some characteristic parameters of the vehicles (such as mechanical resistance, mass and air resistance).

Introducing the third-order equation, the dynamic model is expressed as:

$$\dot{a}_n(t) = \frac{1}{\varsigma_n} u_n(t) - \frac{1}{\varsigma_n} a_n(t) \tag{5}$$

Therefore, the state equation of the third-order model of the nth vehicle is:

$$\dot{x}_n(t) = A_n x_n(t) + B_n u_n(t) \tag{6}$$

where

$$x_n(t) = [p_n(t), v_n(t), a_n(t)]^{\mathrm{T}}$$

$$A_n = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1/\varsigma_n \end{bmatrix}, \quad B_n = \begin{bmatrix} 0 \\ 0 \\ 1/\varsigma_n \end{bmatrix}$$

The distributed cooperative control can make vehicles drive synergistically according to the information of adjacent vehicles and traffic signals. Many studies considered the distributed control strategies for specific topologies. This paper only considers the information impact of ahead vehicles and ignores the information impact of following vehicles. The general form of the linearized controller is expressed as:

$$\begin{aligned}u_n(t) &= \sum_{i=1}^{N_n} [k_p(p_i(t) - p_n(t) - d_{ni})\\ &\quad + k_v(v_i(t) - v_n(t)) + k_a(a_i(t) - a_n(t))]\end{aligned} \tag{7}$$

where N_n represents the set of vehicles cooperating with vehicle n. $k_\#(\# = p, v, a)$ represent the gains of local controller corresponding to position difference, velocity difference and acceleration difference.

Further considering communication delays between vehicles, the expression of the distributed controller is:

$$\begin{aligned}u_n(t - \tau_c) &= \sum_{i=1}^{N_n} [k_p(p_i(t - \tau_c) - p_n(t) - d_{ni})\\ &\quad + k_v(v_i(t - \tau_c) - v_n(t))\\ &\quad + k_a(a_i(t - \tau_c) - a_n(t))]\end{aligned} \tag{8}$$

In order to simplify the mathematical description, the modified formulas (7) and (8) are expressed, respectively, as:

المنارة للاستشارات
www.manaraa.com

$$u_n(t) = \sum_{i=1}^{N_n} k[x_i(t) - x_n(t)] \tag{9}$$

and

$$u_n(t-\tau_c) = \sum_{i=1}^{N_n} k[x_i(t-\tau_c) - x_n(t)] \tag{10}$$

where $k = [k_p, k_v, k_a]$.

The control goal of cooperative driving system is to track the recommended velocity and maintain the formation control strategy of the desired distance between vehicles:

$$\begin{cases} 0 = v_n(t) - v_n^e(t) \\ d_{n-1,n} = p_{n-1}(t) - p_n(t) \\ 0 = a_n(t) - a_n^e(t) \end{cases} \tag{11}$$

where $d_{n-1,n}$ is the desired distance between vehicles $n-1$ and n. $v_n^e(t)$ and $a_n^e(t)$ are the desired velocity and the desired acceleration, respectively.

The relationship of the first vehicle is satisfied as:

$$\dot{x}_0(t) = A_0 x_0(t) + B_0 u_0(t-\tau_c) \tag{12}$$

The desired trajectory of vehicle n is:

$$p_n^e(t) = p_0(t) - d_{0,n} = p_0(t) - \sum_{i=0}^{n-1} d_{i,i+1} \tag{13}$$

Define three new tracking errors $\tilde{p}_n(t), \tilde{v}_n(t), \tilde{a}_n(t)$ as:

$$\tilde{p}_n(t) = p_n(t) - p_n^e(t) \tag{14}$$

$$\tilde{v}_n(t) = v_n(t) - v_n^e(t) \tag{15}$$

$$\tilde{a}_n(t) = a_n(t) - a_n^e(t) \tag{16}$$

So, we obtain

$$u_n(t) = k \sum_{i=1}^{N_n} [\tilde{x}_i(t) - \tilde{x}_n(t)] \tag{17}$$

Assume vector

$$\tilde{x}_n(t) = [\tilde{p}_n(t), \tilde{v}_n(t), \tilde{a}_n(t)]^{\mathrm{T}} \tag{18}$$

The error dynamics system of the nth vehicle is expressed as:

$$\dot{\tilde{x}}_n(t) = A_n \tilde{x}_n(t) + B_n u_n(t) \tag{19}$$

Considering cooperative driving system consisting of N vehicles, the expression of the state vector is obtained as:

$$\tilde{x}(t) = [\tilde{x}_1(t), \tilde{x}_2(t), \ldots, \tilde{x}_N(t)]^{\mathrm{T}} \tag{20}$$

The control input vector is constructed as:

$$u(t) = [u_1(t), u_2(t), \ldots, u_N(t)]^{\mathrm{T}} \tag{21}$$

The measured output vector is constructed as:

$$y(t) = [y_1(t), y_2(t), \ldots, y_N(t)]^{\mathrm{T}} \tag{22}$$

Based on Eqs. (5) and (6), the state space equation of whole cooperative driving system can be written as:

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu(t) \tag{23}$$

where

$$A = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \ldots & \ddots & \ddots & \ldots \\ 0 & \cdots & 0 & A_N \end{bmatrix},$$

$$B = \begin{bmatrix} -B_1 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ B_2 & -B_2 & 0 & \ddots & 0 & 0 & \cdots & 0 \\ B_3 & B_3 & -2B_3 & \ddots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ B_{N_n} & B_{N_n} & B_{N_n} & \cdots & -(N_n-1)B_{N_n} & 0 & \cdots & 0 \\ 0 & B_{N_n} & B_{N_n} & \cdots & B_{N_n} & -(N_n-1)B_{N_n+1} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & B_{N_n} & B_{N_n} & \cdots & B_{N_n} & B_{N_n} & -(N_n-1)B_N \end{bmatrix}$$

المنارة للاستشارات
www.manaraa.c

The output equation is:

$$y(t) = C\tilde{x}(t) \tag{24}$$

where

$$C = \begin{bmatrix} C_1 & 0 & \cdots & 0 \\ 0 & C_2 & \cdots & 0 \\ \cdots & \ddots & \ddots & \cdots \\ 0 & \cdots & 0 & C_N \end{bmatrix}, C_n = I_{3\times 3}.$$

In practical traffic, the vehicles may encounter various uncertain external disturbances in the driving process, such as wind change, road irregularity, equipment failure, etc. Therefore, it is necessary to consider the impact of these uncertainties. $w_n(t)$ represents the combined equivalent disturbance of acceleration and wind drag. In addition, we are interested in the uncertainty of engine inertia delay, which denotes as $\Delta\varsigma_n$.

Considering disturbance inputs, communication delays and parameter uncertainties, the vehicle dynamics model of the nth vehicle could be formulated as:

$$\begin{aligned} \dot{a}_n(t) = &\left(\frac{1}{\varsigma_n} + \frac{1}{\Delta\varsigma_n}\right) u_n(t-\tau_c) \\ &-\left(\frac{1}{\varsigma_n} + \frac{1}{\Delta\varsigma_n}\right) a_n(t) \\ &+ b_{w,n} w_n(t) \end{aligned} \tag{25}$$

$|\Delta\varsigma_n| = l_n(t)$ belongs to the measurable Lebesgue continuous function, which satisfies $l_n^2(t) \geq D_n > 0$, $b_{w,n} = 1/\varsigma_n$.

Therefore, the state equation of the third-order dynamical model is formulated as:

$$\begin{aligned} \dot{x}_n(t) = &(A_n + \Delta A_n)x_n(t) + (B_n + \Delta B_n)u_n(t-\tau_c) \\ &+ b_{w,n} w_n(t) \end{aligned} \tag{26}$$

Based on the formula (26), the state space equation of cooperative driving system can be written as:

$$\dot{\tilde{x}}(t) = (A+\Delta A)\tilde{x}(t) + (B+\Delta B)u(t-\tau_c) + B_w w(t) \tag{27}$$

where

$$B_w = \mathrm{diag}\{b_{w,1}, b_{w,2}, \ldots, b_{w,N}\},$$
$$\Delta A = \mathrm{diag}\{\Delta A_1, \Delta A_2, \ldots, \Delta A_N\},$$
$$\Delta B = \begin{bmatrix} -\Delta B_1 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \Delta B_2 & -\Delta B_2 & 0 & \ddots & 0 & 0 & \cdots & 0 \\ \Delta B_3 & \Delta B_3 & -2\Delta B_3 & \ddots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Delta B_{N_n} & \Delta B_{N_n} & \Delta B_{N_n} & \cdots & -(N_n-1)\Delta B_{N_n} & 0 & \cdots & 0 \\ 0 & \Delta B_{N_n} & \Delta B_{N_n} & \cdots & \Delta B_{N_n} & -(N_n-1)\Delta B_{N_n+1} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \Delta B_{N_n} & \Delta B_{N_n} & \cdots & \Delta B_{N_n} & \Delta B_{N_n} & -(N_n-1)\Delta B_N \end{bmatrix},$$
$$\Delta A_n = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/\Delta\varsigma_n \end{bmatrix}, \Delta B_n = \begin{bmatrix} 0 \\ 0 \\ 1/\Delta\varsigma_n \end{bmatrix}.$$

According to the uncertainty parameter $\Delta\varsigma_n$ hypothesis, we get

$$[\,\Delta A \;\; \Delta B\,] = DL(t)[\,E_1 \;\; E_2\,] \tag{28}$$

where

$$D = \frac{1}{\sqrt{D_n}}$$
$$\mathrm{diag}\left\{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \ldots, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}\right\},$$
$$E_1 = \mathrm{diag}\left\{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ldots, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right\},$$
$$E_2 = \mathrm{diag}\left\{\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}, \right.$$

المنارة للاستشارات
www.manaraa.c

$$\begin{bmatrix}0&0&0\\0&0&0\\1&1&-2\end{bmatrix}, \ldots, \begin{bmatrix}0&0&\cdots&0\\0&0&\cdots&\vdots\\1&1&\cdots&-(n-1)\end{bmatrix},$$
$$\left.\left.\begin{bmatrix}0&0&\cdots&0&\cdots&0\\0&0&\cdots&0&\cdots&\vdots\\0&\cdots&\cdots&1&\cdots&-(n-1)\end{bmatrix}\right]\right\}$$

Besides, the following relationships are also satisfied as:

$$L(t) = \mathrm{diag}\{\sqrt{D_1}/l_1(t), \ldots, \sqrt{D_N}/l_N(t)\}I,$$
$$L^{\mathrm{T}}(t)L(t) \le I \tag{29}$$

2.2 The effect of communication constraints

When using V2X communication, there are time delays in the control loop because the information must be processed by both the senders and the receivers. Firstly, the senders sample their own motion characteristics (GPS positioning, wheels speed); then process the information and send data packets via the wireless protocol IEEE 802.11p on wireless devices such as DSRC (dedicated short-range communications) [33]. If the packets are received successfully by the receivers, the controllers assign the appropriate operating commands to the engine or braking torque according to traffic information. Note that, according to the IEEE 802.11p protocol, packets sent successfully are not received by the receivers, and no packets are resent in dynamic traffic environment. To simplify, we assume that the vehicles clocks are synchronized. In this section, it is assumed that the feedback variables $y(t)$ of the vehicles are measurable. In addition, the output part containing the information of preceding vehicles is quantified and then transmits to following vehicles shown in Fig. 1b. We are interested in the log information quantizer defined in the studies [33,34]. This quantizer is indicated as $f(\cdot)$ and satisfies symmetry, that is, $f(-v) = -f(v)$. For each quantizer $f(\cdot)$, the set of quantized levels is denoted as:

$$U = [\pm\vartheta_n, n = 0, \pm 1, \pm 2, \ldots] \tag{30}$$

where $\vartheta_n = \rho^n \vartheta_0, 0 < \rho < 1, \vartheta_0 > 0$.

Each quantization level corresponds to a piecewise function so that the quantizer maps the entire segment to this quantization level set. Define log information quantizer $f(\cdot)$ as:

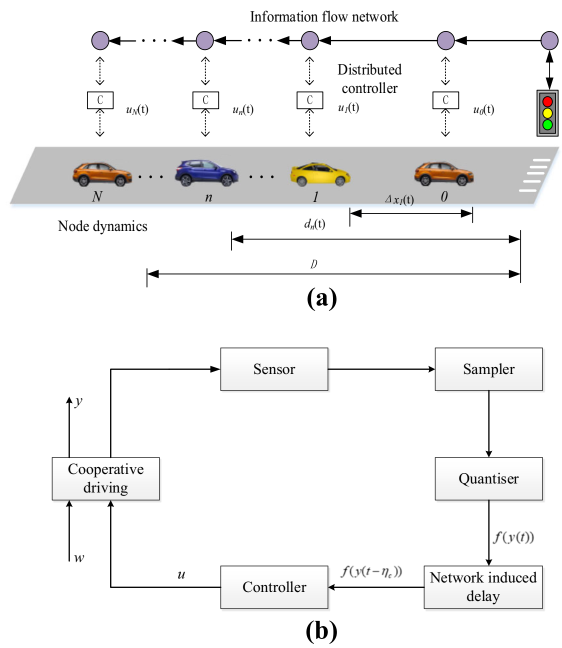

Fig. 1 **a** Cooperative driving model of heterogeneous vehicles in the vicinity of traffic signals [32]; **b** the effect of communication constraints

$$f(y) = \begin{cases} \vartheta_n & \text{if } \frac{1}{1+\sigma} < y < \frac{1}{1-\sigma} \\ 0 & \text{if } y = 0 \\ -f(-y) & \text{if } y < 0 \end{cases} \tag{31}$$

where $\sigma = \frac{1-\rho}{1+\rho}$.

Therefore, the expression of the controller output $u(t)$ is expressed as:

$$u(t) = Kf(y) \tag{32}$$

If the following vehicles can receive the information of preceding vehicles successfully, there is communication delay τ_c which boundary is $\tau_c \le \tau_m$. If information transmission fails, the number of lost packets is expressed as ψ, and the boundary is $\psi \le \psi_m$. The packet loss is also considered as a special time delay ψh, where h is the sampling period. We rearrange the expression (32) and get

$$u(t) = Kf(y(t - \tau_c - \psi h)) \tag{33}$$

Considering the process of information quantification (33), according to the literature [35], the output feedback controller can be obtained as:

$$u(t) = K(1 + \varphi(t))y(t - \eta_c) \tag{34}$$

where $\eta_c = \tau_c + \psi h, \varphi(t) \in [-\sigma, \sigma]$ represents quantization error and takes it as parameter uncertainty.

المنارة للاستشارات
www.manaraa.com

Finally, substituting Eq. (34) into Eq. (27), we obtain cooperative driving model as follows:

$$\dot{\tilde{x}}(t) = (A + \mathrm{DFE}_1)\tilde{x}(t) + (B + \mathrm{DFE}_2)u(t - \eta_c) + B_w w(t) \quad (35)$$

Further, Eq. (35) could be simplified as:

$$\dot{\tilde{x}}(t) = (A + \mathrm{DFE}_1)\tilde{x}(t) + [\mathrm{BKC}(1+\varphi) + \mathrm{DFE}_2\mathrm{KC}(1+\varphi)]\tilde{x}(t - \eta_c) + B_w w(t) \quad (36)$$

Based on the above model, robust stabilization and robust controller are analyzed in the following sections. Next, we need to introduce two lemmas for later discussion.

Lemma 1 (Schur complement) *For a given symmetric matrix* [36]:

$$S^{\mathrm{T}} = \begin{bmatrix} S_{11} & S_{12} \\ * & -S_{22} \end{bmatrix}$$

where $S_{11} \in R^{n\times n}$, *the following conditions are equivalent:*

1. $S < 0$;
2. $S_{11} < 0,\ S_{22} - S_{12}^{\mathrm{T}} S_{11}^{-1} S_{12} < 0$;
3. $S_{22} > 0,\ S_{11} + S_{12} S_{22}^{-1} S_{21} < 0$.

Lemma 2 *For given matrices* $Q = Q^{\mathrm{T}}$, H *and* E *with appropriate dimensions, the matrix inequality* [37]

$$Q + HF(t)E + E^{\mathrm{T}}F^{\mathrm{T}}(t)H^{\mathrm{T}} < 0$$

holds for all $F(t)$ *satisfying* $F^{\mathrm{T}}(t)F(t) \le I$ *if and only if there exists* $\varepsilon > 0$ *such that matrix inequality:*

$$Q + \varepsilon^{-1}HH^{\mathrm{T}} + \varepsilon E^{\mathrm{T}}E < 0$$

3 Robustness analysis of cooperative driving system

3.1 Robust stabilization

In this subsection, robust stabilization analysis will be performed for cooperative driving system (36) with $w(t) = 0$.

Theorem 1 *Consider closed-loop cooperative driving system* (36) *with* $w(t) = 0$. *If there exist positive definite symmetric matrixes* $P = P^{\mathrm{T}} > 0$, $R = R^{\mathrm{T}} > 0$, *the following LMI is satisfied as:*

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ * & \Phi_{22} \end{bmatrix} < 0 \quad (37)$$

Such that the system (36) *is asymptotically stable, where the asterisk "*" represents symmetry. The other elements are shown as follows:*

$$\Phi_{11} = A^{\mathrm{T}}P + \mathrm{PA} + (\mathrm{DFE}_1)^{\mathrm{T}}P + \mathrm{PDFE}_1 + (\mathrm{KC})^{\mathrm{T}}R(\mathrm{KC}),$$
$$\Phi_{12} = (1+\varphi)\mathrm{PB} + (1+\varphi)\mathrm{PDFE}_2,\ \Phi_{22} = -R.$$

Proof The Lyapunov–Krasovskii functional is constructed for cooperative driving system (36), and the expression is shown as follows:

$$V(t) = \tilde{x}^{\mathrm{T}}(t)P\tilde{x}(t) + \int_{t-\eta_c}^{t} \tilde{x}^{\mathrm{T}}(s)(\mathrm{KC})^{\mathrm{T}}\mathrm{RKC}\tilde{x}(s)\mathrm{d}s \quad (38)$$

The derivative $V(t)$ along the trajectory of dynamics Eq. (36) can be obtained as:

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) \quad (39)$$

For the derivative with respect to $V_1(t)$, we get

$$\begin{aligned}\dot{V}_1 &= \dot{\tilde{x}}^{\mathrm{T}}(t)P\tilde{x}(t) + \tilde{x}^{\mathrm{T}}(t)P\dot{\tilde{x}}(t)\\ &= \tilde{x}^{\mathrm{T}}(t)A^{\mathrm{T}}P\tilde{x}(t) + \tilde{x}^{\mathrm{T}}(t)(\mathrm{DFE}_1)^{\mathrm{T}}P\tilde{x}(t)\\ &\quad + \tilde{x}^{\mathrm{T}}(t-\eta_c)(\mathrm{KC})^{\mathrm{T}}(1+\varphi)(B)^{\mathrm{T}}P\tilde{x}(t)\\ &\quad + \tilde{x}^{\mathrm{T}}(t-\eta_c)(\mathrm{KC})^{\mathrm{T}}(1+\varphi)(\mathrm{DFE}_2)^{\mathrm{T}}P\tilde{x}(t)\\ &\quad + \tilde{x}^{\mathrm{T}}(t)\mathrm{PA}\tilde{x}(t) + \tilde{x}^{\mathrm{T}}(t)\mathrm{PDFE}_1\tilde{x}(t)\\ &\quad + \tilde{x}^{\mathrm{T}}(t)(1+\varphi)\mathrm{PBKC}\tilde{x}(t-\eta_c)\\ &\quad + \tilde{x}^{\mathrm{T}}(t)(1+\varphi)\mathrm{PDFE}_2\mathrm{KC}\tilde{x}(t-\eta_c)\end{aligned} \quad (40)$$

For the derivative with respect to $V_2(t)$, we get

$$\dot{V}_2(t) = \tilde{x}^{\mathrm{T}}(t)(\mathrm{KC})^{\mathrm{T}}R(\mathrm{KC})\tilde{x}(t) - \tilde{x}^{\mathrm{T}}(t-\eta_c)(\mathrm{KC})^{\mathrm{T}}R(\mathrm{KC})\tilde{x}(t-\eta_c) \quad (41)$$

Substituting Eqs. (27), (40) and (41) into Eq. (39), we can be obtained as:

$$\dot{V}(t) = \xi^{\mathrm{T}}(t)\Phi\xi(t) \quad (42)$$

where $\xi(t) = [\tilde{x}(t)^{\mathrm{T}}, (\mathrm{KC}\tilde{x}(t-\eta_c))^{\mathrm{T}}]^{\mathrm{T}}$. This completes the proof. □

Remark 1 Theorem 1 provides sufficient conditions for the asymptotically robust stability of cooperative driving model (36), which means that the operation of each vehicle is asymptotically stable. Next, a robust controller is presented.

المنارة للاستشارات

www.manaraa.com

3.2 Robust control

Considering practical driving process, it is necessary to introduce the external disturbances and analyze the robust control strategy of each vehicle to restrain the influence of uncertain external disturbances. Assume that external disturbances $w(t)$ are finite energy function.

Next, we will investigate cooperative driving model with communication delays and external disturbances as follows:

$$\dot{\tilde{x}}(t) = (A + \mathrm{DFE}_1)\tilde{x}(t) + [\mathrm{BKC}(1+\varphi) \\ + \mathrm{DFE}_2\mathrm{KC}(1+\varphi)]\tilde{x}(t-\eta_c) + B_w w(t) \quad (43)$$

For a given H_∞ disturbance attenuation level $\gamma > 0$, the following condition should be met:

$$\left(\int_0^\infty y^2(t)\mathrm{d}t\right)^{1/2} \leq \gamma \left(\int_0^\infty w^2(t)\mathrm{d}t\right)^{1/2} \quad (44)$$

Theorem 2 *For cooperative driving system (43) and the given constant $\gamma > 0$, if there exist positive definite symmetric matrixes R, P, such that the following LMI holds:*

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & C^{\mathrm{T}} \\ * & \Phi_{22} & 0 & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (45)$$

For all the external perturbations allowed, the proposed system (43) is asymptotically robust stable with H_∞ disturbance attenuation level γ. The output feedback controller is:

$$u(t) = K(1+\varphi)y(t-\eta_c) \quad (46)$$

where

$$\Phi_{11} = A^{\mathrm{T}}P + PA + (DFE_1)^{\mathrm{T}}P \\ +PDFE_1 + (KC)^{\mathrm{T}}R(KC),$$

$$\Phi_{12} = (1+\varphi)PB + (1+\varphi)PDFE_2,$$

$$\Phi_{22} = -R,$$

$$\Phi_{13} = PB_w$$

Proof The Lyapunov–Krasovskii functional is constructed for cooperative driving system (43), and the expression is as follows:

$$V(t) = \tilde{x}^{\mathrm{T}}(t)P\tilde{x}(t) + \int_{t-\eta_c}^{t} \tilde{x}^{\mathrm{T}}(s)(\mathrm{KC})^{\mathrm{T}}R(\mathrm{KC})\tilde{x}(s)\mathrm{d}s \quad (47)$$

The derivative $V(t)$ along the trajectory of dynamics equation (43) can be obtained as:

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) \quad (48)$$

For the derivative with respect to $V_1(t)$, we get

$$\begin{aligned} \dot{V}_1 &= \dot{\tilde{x}}^{\mathrm{T}}(t)P\tilde{x}(t) + \tilde{x}^{\mathrm{T}}(t)P\dot{\tilde{x}}(t) \\ &= \tilde{x}^{\mathrm{T}}(t)A^{\mathrm{T}}P\tilde{x}(t) + \tilde{x}^{\mathrm{T}}(t)(\mathrm{DFE}_1)^{\mathrm{T}}P\tilde{x}(t) \\ &\quad +\tilde{x}^{\mathrm{T}}(t-\eta_c)(\mathrm{KC})^{\mathrm{T}}(1+\varphi)B^{\mathrm{T}}P\tilde{x}(t) \\ &\quad +w^{\mathrm{T}}(t)B_w P\tilde{x}(t) \\ &\quad +\tilde{x}^{\mathrm{T}}(t-\eta_c)(\mathrm{KC})^{\mathrm{T}}(1+\varphi)(\mathrm{DFE}_2)^{\mathrm{T}}P\tilde{x}(t) \\ &\quad +\tilde{x}^{\mathrm{T}}(t)\mathrm{PA}\tilde{x}(t) + \tilde{x}^{\mathrm{T}}(t)\mathrm{PDFE}_1\tilde{x}(t) \\ &\quad +\tilde{x}^{\mathrm{T}}(t)(1+\varphi)\mathrm{PBKC}\tilde{x}(t-\eta_c) \\ &\quad +\tilde{x}^{\mathrm{T}}(t)(1+\varphi)\mathrm{PDFE}_2\mathrm{KC}\tilde{x}(t-\eta_c) \\ &\quad +\tilde{x}^{\mathrm{T}}(t)\mathrm{PB}_w w(t) \end{aligned} \quad (49)$$

For the derivative with respect to $V_2(t)$, we get

$$\begin{aligned} \dot{V}_2(t) &= \tilde{x}^{\mathrm{T}}(t)(\mathrm{KC})^{\mathrm{T}}R(\mathrm{KC})\tilde{x}(t) \\ &\quad -\tilde{x}^{\mathrm{T}}(t-\eta_c)(\mathrm{KC})^{\mathrm{T}}R(\mathrm{KC})\tilde{x}(t-\eta_c) \end{aligned} \quad (50)$$

Substituting Eqs. (43), (50) and (49) into Eq. (48), we can be obtained as:

$$\dot{V}(t) = \xi^{\mathrm{T}}(t)\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ * & \Phi_{22} & 0 \\ * & * & 0 \end{bmatrix}\xi(t) \quad (51)$$

where $\xi(t) = [\tilde{x}(t)^{\mathrm{T}}, \tilde{x}(t-\eta_c)^{\mathrm{T}}(\mathrm{KC})^{\mathrm{T}}, w(t)^{\mathrm{T}}]^{\mathrm{T}}$.

Next, assume the initial condition is zero and introduce

$$J = \int_0^\infty [y^{\mathrm{T}}(t)y(t) - \gamma^2 w^{\mathrm{T}}(t)w(t)]\mathrm{d}t \quad (52)$$

Therefore, for any nonzero $w(t) \in L_2[0, \infty)$, we have

$$\begin{aligned} J &\leq \int_0^\infty [y^{\mathrm{T}}(t)y(t) - \gamma^2 w^{\mathrm{T}}(t)w(t) + \dot{V}(t)]\mathrm{d}t \\ &\leq \int_0^\infty \xi^{\mathrm{T}}(t)\tilde{\Phi}\xi(t)\mathrm{d}t \end{aligned} \quad (53)$$

where

$$\tilde{\Phi} = \begin{bmatrix} \Phi_{11} + C^{\mathrm{T}}C & \Phi_{12} & \Phi_{13} \\ * & \Phi_{22} & 0 \\ * & * & -\gamma^2 I \end{bmatrix}$$

When $\tilde{\Phi} < 0$, we have $J < 0$. If there exist a matrix K, a positive definite matrix $Q_1 = Q_1^{\mathrm{T}} > 0$ and a positive scalar $\gamma > 0$, such that the following linear matrix inequality $\tilde{\Phi} < 0$ holds, then the system (43) is robustly stable. Applying Lemma 1 (Schur complement), we can obtain the inequation (45). This completes the proof. □

المنارة للاستشارات
www.manaraa.com

We further simplify the linear matrix inequality (45).

Theorem 3 *For cooperative driving system* (43) *and the given constant* $\gamma > 0$*, if there exist positive definite symmetric matrixes* X*,* S*, a matrix* M *and the scalars* $\varepsilon_1, \varepsilon_2, \varepsilon_3 > 0$*, such that*

$$\begin{bmatrix} \Pi_3 & 0 & 0 & 0 & B_w & (CQ)^{\mathrm{T}} & M^{\mathrm{T}} & (E_1Q)^{\mathrm{T}} \\ * & -S & S & SE_2^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & * & -I\varepsilon_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I\varepsilon_3 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & * & -I\varepsilon_1 \end{bmatrix} < 0 \tag{54}$$

holds, then the system (43) *is robustly stable with* H_∞ *norm bound* γ *by controller* (46)*. Here, some terms are defined as follows:*

$$X = P^{-1}, M = KCX, S = R^{-1},$$
$$\Pi_3 = AX + (AX)^{\mathrm{T}} + (\varepsilon_1 + 4\varepsilon_3)DD^{\mathrm{T}} + 4\varepsilon_2 BB^{\mathrm{T}}.$$

Proof The following proof is mainly completed by Schur supplement and variable transformation. The inequality (45) is equivalent to

$$\begin{bmatrix} \begin{matrix} A^{\mathrm{T}}P + \mathrm{PA} \\ +(\mathrm{DFE}_1)^{\mathrm{T}}P \\ +\mathrm{PDFE}_1 + (\mathrm{KC})^{\mathrm{T}}R(\mathrm{KC}) \end{matrix} & \begin{matrix} (1+\varphi)\mathrm{PB} \\ +(1+\varphi)\mathrm{PDFE}_2 \end{matrix} & \mathrm{PB}_w & C^{\mathrm{T}} \\ * & -R & 0 & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{55}$$

Using Lemma 2 for transformation, it can be obtained as:

$$\begin{bmatrix} \Pi_1 & 0 & \mathrm{PB}_w & C^{\mathrm{T}} \\ * & -R + \varepsilon_2^{-1} + E_2^{\mathrm{T}}\varepsilon_3^{-1}E_2 & 0 & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{56}$$

where

$$\Pi_1 = A^{\mathrm{T}}P + \mathrm{PA} + (\mathrm{KC})^{\mathrm{T}}R(\mathrm{KC}) + (\varepsilon_1 + 4\varepsilon_3)\mathrm{PDD}^{\mathrm{T}}P$$
$$+4\varepsilon_2\mathrm{PBB}^{\mathrm{T}}P + \varepsilon_1^{-1}E_1^{\mathrm{T}}E.$$

Using Schur complement again, we get

$$\begin{bmatrix} \Pi_2 & 0 & 0 & 0 & \mathrm{PB}_w & C^{\mathrm{T}} & (\mathrm{KC})^{\mathrm{T}} & E_1^{\mathrm{T}} \\ * & -R & I & E_2^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & * & -I\varepsilon_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I\varepsilon_3 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -R^{-1} & 0 \\ * & * & * & * & * & * & * & -I\varepsilon_1 \end{bmatrix} < 0 \tag{57}$$

Multiply both sides of the inequality (57) by the following matrix:

$$\begin{bmatrix} P^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & R^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & I & 0 & 0 & 0 & 0 \\ * & * & * & * & I & 0 & 0 & 0 \\ * & * & * & * & * & I & 0 & 0 \\ * & * & * & * & * & * & I & 0 \\ * & * & * & * & * & * & * & I \end{bmatrix}$$

Then, the inequality (57) can be simplified as:

$$\begin{bmatrix} \Pi_3 & 0 & 0 & 0 & B_w & (CX)^{\mathrm{T}} & M^{\mathrm{T}} & (E_1X)^{\mathrm{T}} \\ * & -S & S & SE_2^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ * & * & -I\varepsilon_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I\varepsilon_3 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & * & -I\varepsilon_1 \end{bmatrix} < 0 \tag{58}$$

To solve the feedback control gain K, we write $X = \mathrm{diag}\{\hat{X}, \hat{X}, \ldots, \hat{X}\}$, and

$$M = \begin{bmatrix} \hat{M} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \hat{M} & \hat{M} & 0 & \ddots & 0 & 0 & \cdots & 0 \\ \hat{M} & \hat{M} & \hat{M} & \ddots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{M} & \hat{M} & \hat{M} & \cdots & \hat{M} & 0 & \cdots & 0 \\ 0 & \hat{M} & \hat{M} & \cdots & \hat{M} & \hat{M} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \hat{M} & \hat{M} & \cdots & \hat{M} & \hat{M} & \hat{M} \end{bmatrix}$$

Then, the gains of feedback control are obtained as $K = \mathrm{diag}\{k, k, \ldots, k\}, k = \hat{M}\hat{X}^{-1}$. This completes the proof. □

المنارة للاستشارات
www.manaraa.c

4 Numerical simulations

To verify the effectiveness of the proposed robust control strategies, the proposed method is compared with the existing method proposed by Guo et al. [17] (called Guo method). First, the experiment is set up. Then, simulation experiments are carried out under V2X communication to evaluate the performance of the proposed control strategies.

Set the speed range of vehicles on city roads $v \in [0, 16.66]$, which corresponds to maximum speed $v_{\max} = 60$ km/h. The desired speed of cooperative driving is $v_{\mathrm{d}} = 11.11$ m/s, that is, $v_{\mathrm{d}} = 40$ km/h. In order to ensure that the vehicle does not accelerate or decelerate violently, the acceleration/deceleration range is set in $a \in [-3, 3]$. The traffic signals period is $T_{\mathrm{C}} = 60$ s, where green time is $T_{\mathrm{g}} = 25$ s and red time is $T_{\mathrm{r}} = 35$ s. We mark green light starts flashing $g_{\mathrm{b}} = 0$ s, and ending flashing is $g_{\mathrm{e}} = 25$ s. The research range of the vicinity of traffic signals is $D = 300$ m and communication range is D_c, where $D_c \leq D$. The distance between the vehicle and traffic signals is $d_n(t)$. The position, speed and acceleration of the nth vehicle at time t are $p(t)$, $v_n(t)$ and $a_n(t)$, respectively. The initial distance between the vehicle and the vehicle is 30 m.

When the vehicles are traveling in the vicinity of traffic signals, the vehicles provide two options: (1) If the vehicles cannot pass during green light, they need to stop at the parking line; (2) if it passes during green light, the vehicles can travel synergistically with desired speed. The track of vehicles movement in the vicinity of traffic signals can be summarized into four typical modes as shown in Fig. 2.

When the vehicles enter the vicinity of traffic signals, the phase of traffic signals is red. The remaining time of red signals is assumed to be $t_{\mathrm{r\_rest}}$.

The vehicles can make four decisions as follows:

1. If vehicles run with constant speed and occur the violation of red light, they need to stop at stop line

$$\text{if } v_n(t) \cdot t_{\mathrm{r\_rest}} > d_n(t); \quad \text{then } a_t = a_d \tag{59}$$

2. When the vehicles run with constant speed, they will not run red lights

$$\text{if } v_n(t) \cdot t_{\mathrm{r\_rest}} \leq d_n(t); \quad \text{then } a_t = 0 \tag{60}$$

There are two kinds of situations: (3) The vehicles can speed up all the time; (4) the vehicles accelerate

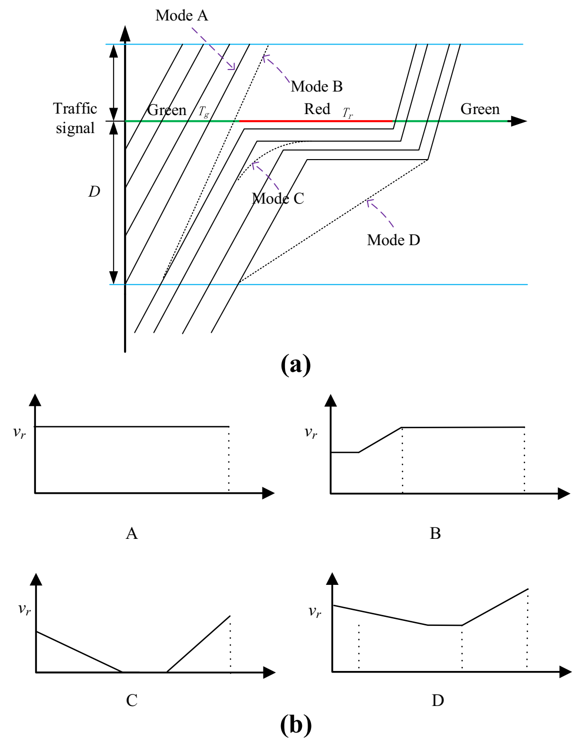

Fig. 2 **a** Vehicle trajectories in the vicinity of traffic signals, **b** four modes of vehicles [31]

firstly and then move at constant speed. The specific expressions hold as follows:

$$\begin{aligned} &\text{if } v_{\max} \geq a_{\max} t_{\mathrm{r\_rest}} + v_n(t) \quad \text{and } d_n(t) \geq v_n(t) t_{\mathrm{r\_rest}} \\ &+ \frac{1}{2} a_{\max} t_{\mathrm{r\_rest}}^2; \quad \text{then } a_t = a_{\max} \end{aligned} \tag{61}$$

$$\begin{aligned} &\text{if } t_a = (v_{\max} - v_n(t))/a_{\max} < t_{\mathrm{r\_rest}} \text{ and} \\ &d_n(t) \geq v_n(t) t_a + \frac{1}{2} a_{\max} t_a^2 + v_{\max}(t_{\mathrm{r\_rest}} - t_a); \\ &\text{then } a_t = a_{\max} \end{aligned} \tag{62}$$

When the vehicles enter the vicinity of traffic signals, the phase of traffic signals is green. The remaining time of green signals is assumed to be $t_{\mathrm{g\_rest}}$. The time required to accelerate the vehicle to its maximum speed is:

$$t_a = \frac{v_{\max} - v_n(t)}{a_{\max}} \tag{63}$$

The nth vehicle can make the following decisions during green signals:

1. The vehicles are moving at a constant speed

$$\text{if } v_n(t) \cdot t_{\mathrm{g\_rest}} \leq d_n(t); \quad \text{then } a_t = 0 \tag{64}$$

www.manaraa.c

2. The vehicles accelerate if the speed is less than the maximum limit speed

$$\begin{aligned}&\text{if } v_{\max} \geq a_{\max} t_{\text{g\_rest}} + v_n(t) \text{ and}\\ &d_n(t) \geq v_n(t) t_{\text{g\_rest}} + \frac{1}{2} a_{\max} t_{\text{g\_rest}}^2;\\ &\text{then } a_t = a_{\max}\end{aligned} \tag{65}$$

3. The vehicles speed up to its maximum speed and then travel at a constant speed

$$\begin{aligned}&\text{if } t_a = (v_{\max} - v_n(t))/a_{\max} < t_{\text{g\_rest}} \text{ and}\\ &d_n(t) \geq v_n(t) t_a + \frac{1}{2} a_{\max} t_a^2 + v_{\max}(t_{\text{g\_rest}} - t_a);\\ &\text{then } a_t = a_{\max}\end{aligned} \tag{66}$$

The vehicles must slow down gently and stop to avoid sudden braking, rear-end collision and red light running, when they cannot pass under green light. We assume that the vehicle takes as little deceleration as possible to slow down, satisfying the following expression:

$$d_n(t) = v_n(t)t + 1/2 a_d t_{\text{g\_rest}}^2 \tag{67}$$

In addition, the following parameters are also used:

1. Inertial delay is $\varsigma_n = 0.5s$, $|\Delta\varsigma_n| = |D_n/\sin(t)|$, $D_n = 4$, and the disturbance attenuation is $\gamma = 1$.
2. Communication delays are $\tau_m = 0, 200\,\text{ms}, 400\,\text{ms}$. One of the three communication delays is randomly selected, with probability of 1/4, 2/4 and 1/4, respectively.
3. Packets loss are $\theta_m = 0, 2, 4$. One of the three packets loss is randomly selected with probability of 1/4, 2/4 and 1/4, respectively. The period is 100 ms. Meanwhile, time delays of all vehicles remain unchanged during each cycle.

Two quantitative indicators are introduced to evaluate the robustness and driving smoothness of speed tracking as follows:

$$\text{MVE} = \max\{|\delta v_i(t)|\}, \quad t \in [t_s, t_e] \tag{68}$$

$$\text{MAE} = \max\{a_i(t)\}, \quad t \in [t_s, t_e] \tag{69}$$

We explain the physical implications of these two performance indicators. The metric MVE is max velocity error in the platoon from the reference speed at time t, indicating the controller's suppression performance of speed pulse, which is used to evaluate speed tracking performance. MAE defines driving smoothness. Generally speaking, driving smoothness is positively correlated with driving comfort. The better the driving smoothness, the higher the driving comfort is. MAE is the maximum acceleration error of all vehicles in the platoon at time t, which represents the suppression performance of the controller's longitudinal acceleration pulse.

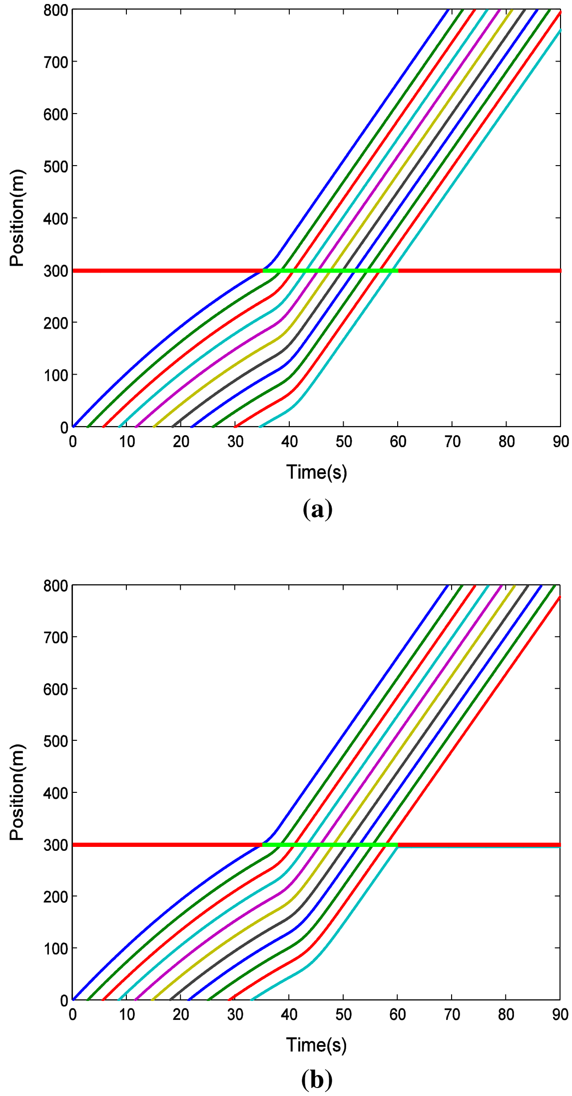

Fig. 3 Vehicle trajectories without disturbance: **a** proposed method, **b** Guo method

In the case of no disturbance, there is a feasible solution $k = [-0.031, -2.623, -1.514]$.

Figure 3 shows the trajectories of vehicles without disturbances. Under the reasonable control strategy, the

www.manaraa.com

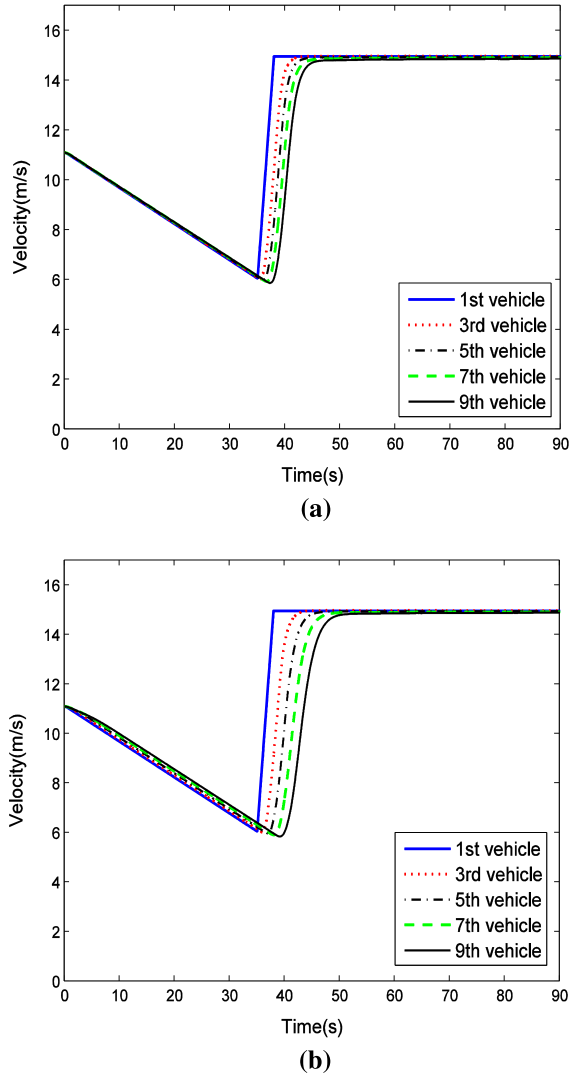

Fig. 4 The velocity tracking curves without disturbance: **a** proposed method, **b** Guo method

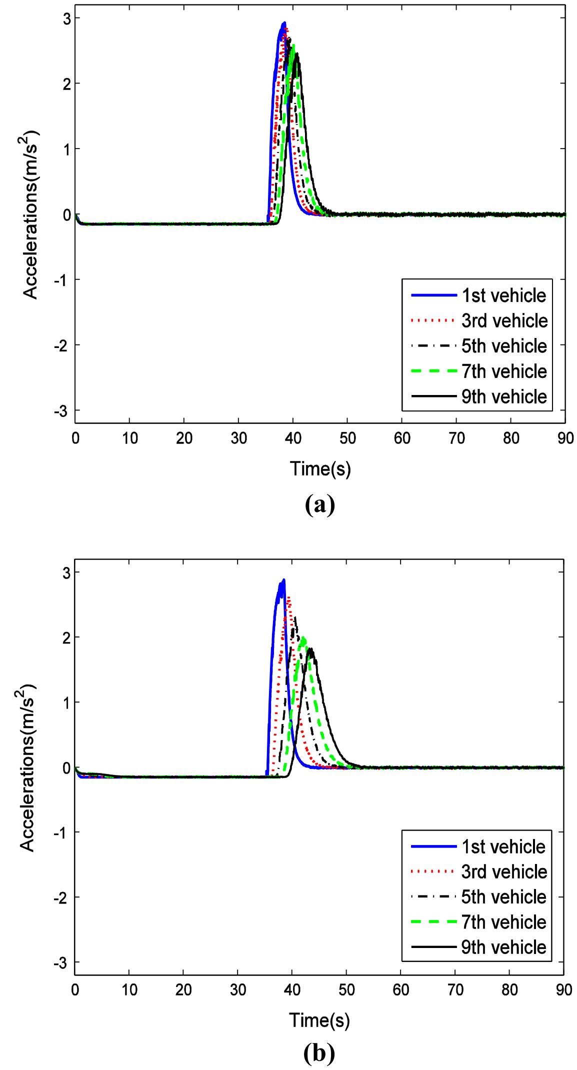

Fig. 5 The acceleration without disturbance: **a** proposed method, **b** Guo method

proposed method does not have the stopping and waiting phenomenon, and all vehicles can pass the vicinity of traffic signals during green light. However, for Guo method, the last vehicle cannot pass green light and is forced to stop at the stop line.

Figure 4 is the speed tracking of vehicles without disturbances. It can be observed that the proposed method can track the speed change rapidly and respond promptly to the impact caused by the change of traffic signals. Guo method has a slow speed tracking response and poor speed tracking performance.

Figure 5 shows the vehicle accelerations without disturbances. It can be seen that the proposed method has relatively consistent acceleration changes, and all vehicles have better synchronous accelerations. The acceleration changes of Guo method are relatively dispersed, due to the poor ability that the following vehicles respond to the changes of the front vehicles.

Figure 6 shows the temporal max velocity error and temporal max acceleration error. From Fig. 6a, it can be observed that the vehicles in the proposed method can quickly track the vehicle in front of it, and the instantaneous maximum speed error between all vehicles is about 1.38 m/s. The instantaneous maximum speed deviation of Guo method is about 2.35 m/s.

المنارة للاستشارات
www.manaraa.c

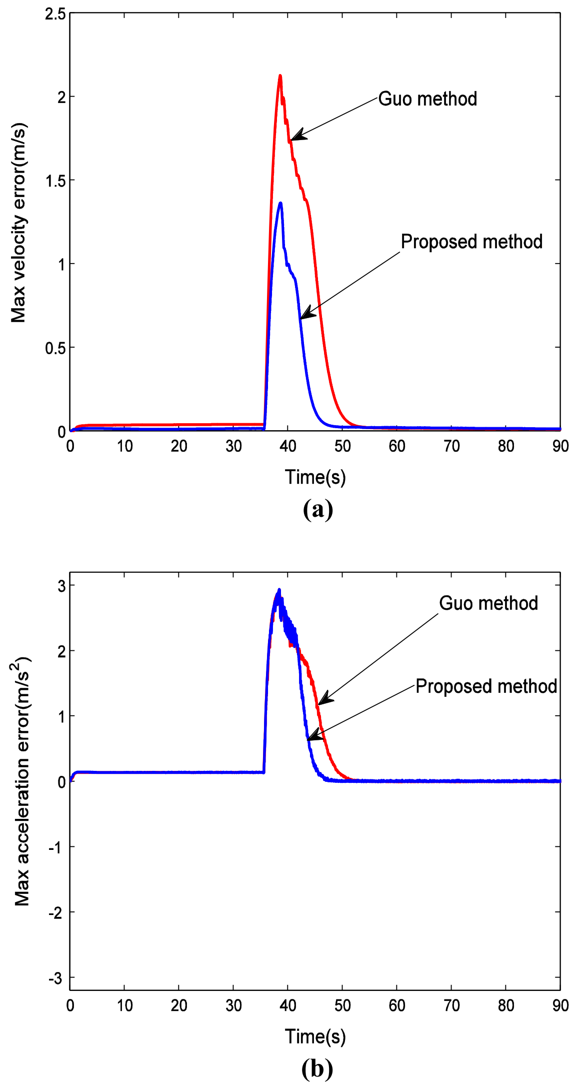

Fig. 6 **a** Temporal max velocity error, **b** temporal max acceleration error

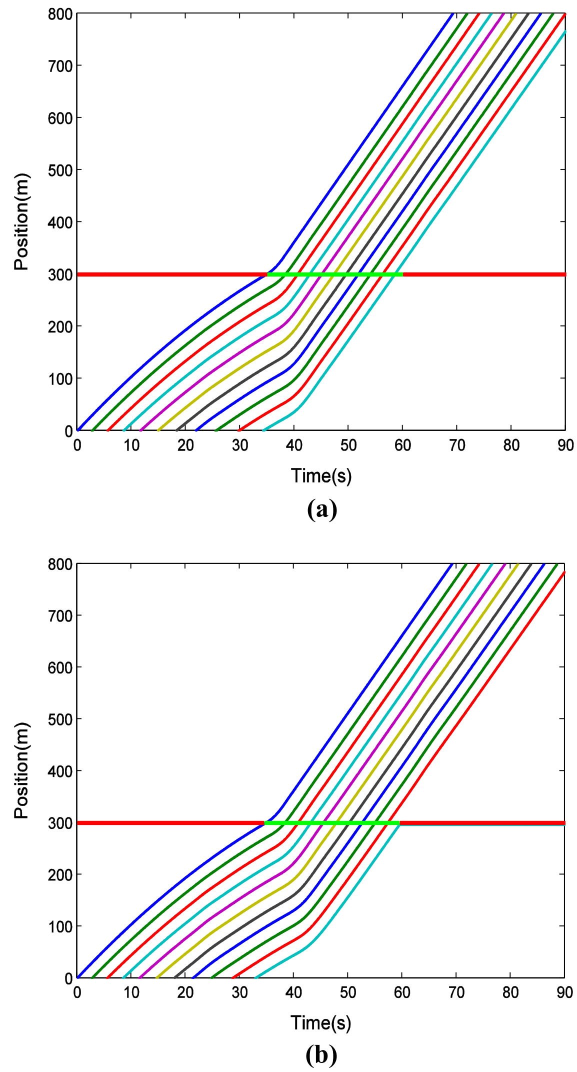

Fig. 7 Vehicle trajectories with disturbance: **a** proposed method, **b** Guo method

By comparison, Fig. 6a indicates that the proposed method has good speed tracking performance. Figure 6b shows that the acceleration changes of the two models are relatively similar, but the response time of Guo method lags a little. So, the proposed method has good driving smoothness. In addition, the accelerations of vehicles fluctuate in a small range due to random communication delay, and the maximum acceleration deviation is within a reasonable range.

Therefore, the proposed method can effectively inhibit traffic congestion and prevent pulse oscillations in speed and acceleration, which leads to improving driving comfort and tracking performance.

Further, in the case of perturbations, there exists a feasible solution to the inequality (5.61) $k = [-0.034, -3.423, -1.842]$.

Figure 7 shows motion trajectories of the vehicles under disturbances. As shown in Fig. 7, the proposed method enables all vehicles to pass through the vicinity of traffic signals during green light, and Guo method cannot guarantee that all vehicles pass during green light and some vehicles stop.

It can be observed from Figs. 8 and 10a that both the proposed method and Guo method can carry out

المنارة للاستشارات
www.manaraa.c

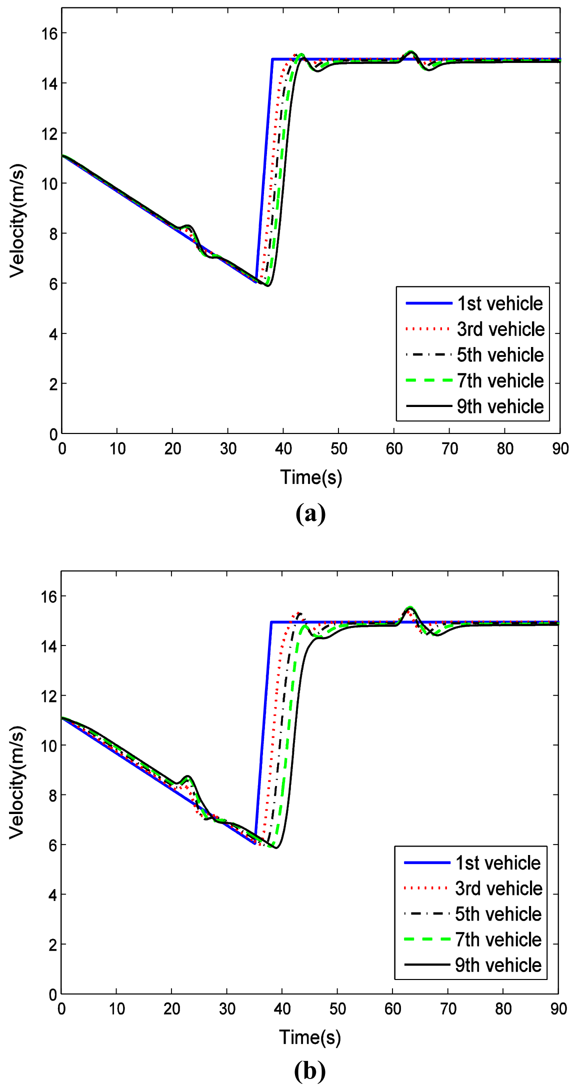

Fig. 8 The velocity tracking curves with disturbance: **a** proposed method, **b** Guo method

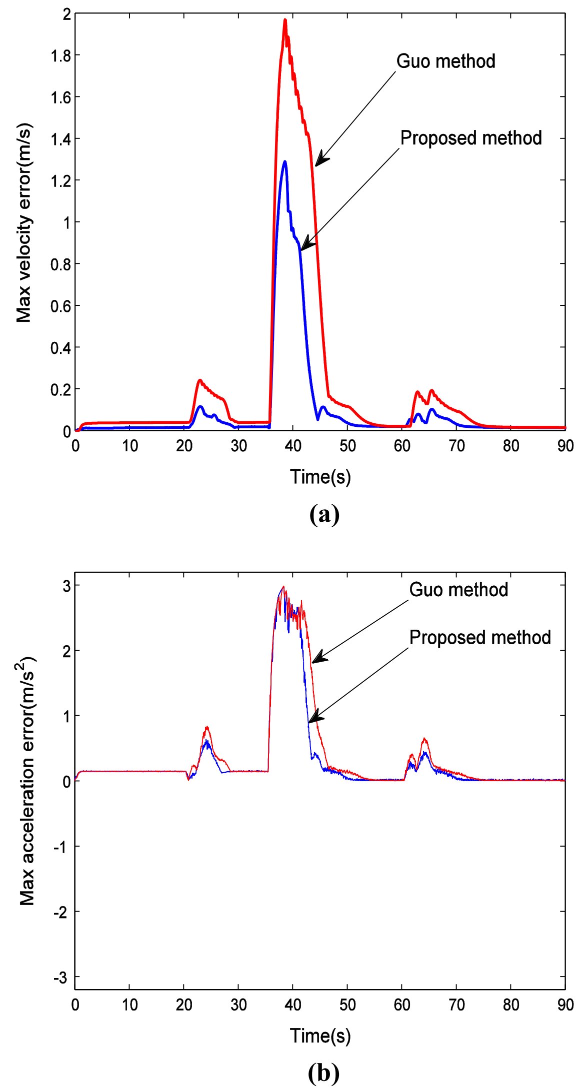

Fig. 9 **a** Temporal max velocity error, **b** temporal max acceleration error

speed tracking without continuous velocity oscillation. However, the vehicles controlled by the proposed method can respond rapidly and track accurately, and the instantaneous maximum speed deviation is about 1.32 m/s, and most of the time is close to 0. The maximum instantaneous speed deviation of Guo method is about 2 m/s. It shows that the proposed method has good tracking effect and no significant change in velocity. For Fig. 10b, compared with Guo method, it can be seen that the proposed method has smaller acceleration changes, faster acceleration response, and good tracking performance and driving smoothness (Fig. 9).

So, the proposed method can effectively suppress traffic congestion, prevent velocity and acceleration oscillations, and improve driving smoothness and tracking performance under external disturbances.

5 Conclusions

Considering the influence of heterogeneity, internal parameter uncertainties and communication constraint, the cooperative driving model is established in the vicinity of traffic signals. By analyzing the robust sta-

المنارة للاستشارات
www.manaraa.c

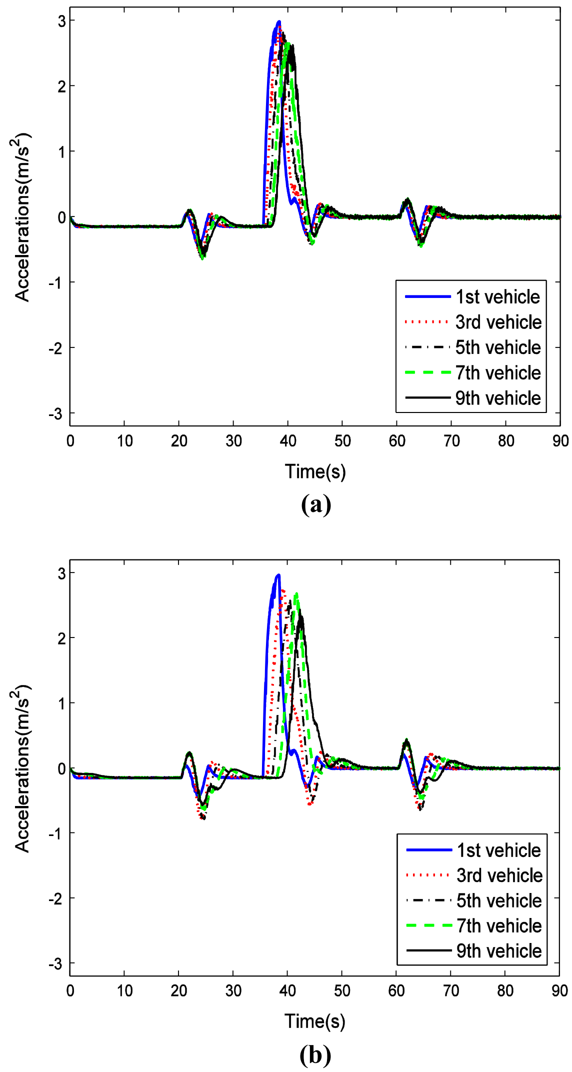

Fig. 10 The acceleration with disturbance: **a** proposed method, **b** Guo method

bility of the proposed model, this paper studies how to maintain the stability and system performance of vehicles cooperative driving dynamics under the interaction of cooperative driving process and traffic information under various internal parameter uncertainties and external disturbances. According to the limitations of heterogeneity, uncertainties, communication delays and packets loss, a robust control strategy is designed. Two quantitative indicators are introduced to evaluate the robustness and smoothness of speed tracking. Through simulation experiments, the proposed method can effectively suppress traffic congestion and prevent pulse oscillations in speed and acceleration, leading to improving tracking performance and comfort. The results show the effectiveness and feasibility of the proposed method and provide guidance for traffic congestion control strategy.

Acknowledgements This work was supported by the National Natural Science Foundation of China (Grant Nos. 61573075, 61803052), the Doctoral Research Project of Guangxi University of Finance and Economics (Grant No. BS2019027), the Foundation for High-level Talents of Chongqing University of Art and Sciences (Grant No. 2017RJD13) and the Natural Science Foundation of Chongqing (Grant No. cstc2017jcyjBX0001).

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Wang, M., Daamen, W., Hoogendoorn, S.P., Arem, B.: Rolling horizon control framework for driver assistance systems. Part II: cooperative sensing and cooperative control. Transp. Res. Part C **40**, 290–311 (2014)
2. Tang, T.Q., Shi, W.F., Shang, H.Y., Wang, Y.P.: A new car-following model with consideration of inter-vehicle communication. Nonlinear Dyn. **76**(4), 2017–2023 (2014)
3. Li, Y.F., Zhao, H., Zheng, T.X., Sun, F.L., Feng, H.Z.: Non-lane-discipline-based car-following model incorporating the electronic throttle dynamics under connected environment. Nonlinear Dyn. **90**(4), 2345–2358 (2017)
4. Avedisov, S.S., Orosz, G.: Analysis of connected vehicle networks using network-based perturbation techniques. Nonlinear Dyn. **89**(3), 1651–1672 (2017)
5. Jia, D.Y., Ngoduy, D.: Enhanced cooperative car-following traffic model with the combination of V2V and V2I communication. Transp. Res. Part B **90**, 172–191 (2016)
6. Jin, Q., Wu, G.Y., Boriboonsomsin, K., Barth, M.J.: Power-based optimal longitudinal control for a connected eco-driving system. IEEE Trans. Intell. Transp. Syst. **17**(10), 2900–2910 (2016)
7. Liu, B.Y., Yao, D.Y., Lu, K.J., Ngoduy, D., Wang, J.P., Wu, L.B.: A joint control-communication design for reliable vehicle platooning in hybrid traffic. IEEE Trans. Veh. Technol. **66**(10), 9394–9409 (2017)
8. Yu, S.W., Liu, Q.L., Li, X.H.: Full velocity difference and acceleration model for a car-following theory. Commun. Nonlinear Sci. Numer. Simul. **18**, 1229–1234 (2013)
9. Jia, D.Y., Ngoduy, D.: Platoon based cooperative driving model with consideration of realistic inter-vehicle communication. Transp. Res. Part C **68**, 245–264 (2016)
10. Jia, D.Y., Lu, K.J., Wang, J.P., Zhang, X., Shen, X.M.: A survey on platoon-based vehicular cyber-physical systems. IEEE Commun. Surv. Tutor. **18**(1), 263–284 (2016)
11. Liu, H., Sun, D.H., Zhao, M.: Analysis of traffic flow based on car-following theory: a cyber-physical perspective. Nonlinear Dyn. **84**(2), 881–893 (2016)

www.manaraa.c

12. Li, Z.P., Li, W.Z., Xu, S.Z., Qian, Y.Q.: Analyses of vehicle's self-stabilizing effect in an extended optimal velocity model by utilizing historical velocity in an environment of intelligent transportation system. Nonlinear Dyn. **80**(1–2), 529–540 (2015)
13. Xue, Y., Guo, Y., Shi, Y., Lv, L.Z., He, H.D.: Feedback control for the lattice hydrodynamics model with drivers' reaction time. Nonlinear Dyn. **88**(1), 145–156 (2017)
14. Li, X.H., Yang, T., Liu, J., Qin, X.Q., Yu, S.W.: Effects of vehicle gap changes on fuel economy and emission performance of the traffic flow in the ACC strategy. PLoS ONE **13**, e0200110 (2018)
15. Al-Jhayyish, A.M.H., Schmidt, K.W.: Feedforward strategies for cooperative adaptive cruise control in heterogeneous vehicle strings. IEEE Trans. Intell. Transp. Syst. **19**(1), 113–122 (2018)
16. Gao, F., Li, S.B., Zheng, Y., Kum, D.: Robust control of heterogeneous vehicular platoon with uncertain dynamics and communication delay. IET Intel. Transp. Syst. **10**(7), 503–513 (2016)
17. Zheng, Y., Li, S.B., Li, K.Q., Ren, W.: Platooning of connected vehicles with undirected topologies: robustness analysis and distributed H-infinity controller synthesis. IEEE Trans. Intell. Transp. Syst. **19**(5), 1353–1364 (2017)
18. Guo, G., Yue, W.: Hierarchical platoon control with heterogeneous information feedback. IET Control Theory Appl. **5**(15), 1766–1781 (2011)
19. Zheng, Y., Li, S.B., Wang, J.Q., Cao, D.P., Li, K.Q.: Stability and scalability of homogeneous vehicular platoon: study on the influence of information flow topologies. IEEE Trans. Intell. Transp. Syst. **17**(1), 14–26 (2016)
20. Li, S.B., Gao, F., Li, K.Q., Wang, L.Y., You, K.Y., Cao, D.P.: Robust longitudinal control of multi-vehicle systems-a distributed h-infinity method. IEEE Trans. Intell. Transp. Syst. **19**(9), 2779–2788 (2018)
21. Li, S.B., Gao, F., Cao, D.P., Li, K.Q.: Multiple-model switching control of vehicle longitudinal dynamics for platoon-level automation. IEEE Trans. Veh. Technol. **65**(6), 4480–4492 (2016)
22. Liu, Y.G., Gao, H.L., Xu, B.G., Liu, G.Y., Cheng, H.: Autonomous coordinated control of a platoon of vehicles with multiple disturbances. IET Control Theory Appl. **8**(18), 2325–2335 (2014)
23. Chang, H.J., Park, G.T.: A study on traffic signal control at signalized intersections in vehicular ad hoc networks. Ad Hoc Netw. **11**(7), 2115–2124 (2013)
24. Asadi, B., Vahidi, A.: Predictive cruise control: utilizing upcoming traffic signal information for improving fuel economy and reducing trip time. IEEE Trans. Control Syst. Technol. **19**(3), 707–714 (2011)
25. Liu, B., Kamel, A.E.: V2X-based decentralized cooperative adaptive cruise control in the vicinity of intersections. IEEE Trans. Intell. Transp. Syst. **17**(3), 644–658 (2016)
26. Yu, S.W., Fu, R., Guo, Y.S., Xin, Q., Shi, Z.K.: Consensus and optimal speed advisory model for mixed traffic at an isolated signalized intersection. Physica A **531**(1), 121789 (2019)
27. Xia, H., Boriboonsomsin, K., Barth, M.: Dynamic eco-driving for signalized arterial corridors and its indirect network-wide energy/emissions benefits. J. Intell. Transp. Syst. **17**(1), 31–41 (2013)
28. Yu, S.W., Shi, Z.K.: Analysis of car-following behaviors considering the green signal countdown device. Nonlinear Dyn. **82**(1–2), 731–740 (2015)
29. Bo, Z., Ge, G., Wang, L.Y., Qiong, W.: Vehicle speed planning and control for fuel consumption optimization with traffic light state. Acta Autom. Sin. **44**(3), 461–470 (2018)
30. Malikopoulos, A.A., Cassandras, C.G., Zhang, Y.J.: A decentralized energy-optimal control framework for connected automated vehicles at signal-free intersections. Automatica **93**, 244–256 (2018)
31. Chen, D., Sun, D.H., Li, Y., Zhao, M., Zheng, L.J.: Robust stabilization and $H\infty$ control of cooperative driving system with time delay in variable speed-limited area from cyber-physical perspective. Asian J. Control **21**(6), 1–15 (2019). https://doi.org/10.1002/asjc.1870
32. Chen, D., Sun, D.H., Zhao, M., Yang, L.Y., Zhou, T., Xie, F.: Distributed robust $H\infty$ control of connected eco-driving system with time-varying delay and external disturbances in the vicinity of traffic signals. Nonlinear Dyn. **92**(4), 1829–1844 (2018)
33. Huang, C.L., Fallah, Y.P., Sengupta, R., Krishnan, H.: Adaptive intervehicle communication control for cooperative safety systems. IEEE Netw. Mag. Global Internetwork. **24**(1), 6–13 (2010)
34. Elia, N., Mitter, S.K.: Stabilization of linear systems with limited information. IEEE Trans. Autom. Control **46**(9), 1384–1400 (2002)
35. Fu, M.Y., Xie, L.H.: The sector bound approach to quantized feedback control. IEEE Trans. Autom. Control **50**(11), 1698–1711 (2005)
36. Boyd, S., Elghaoui, L., Feron, E., Balakrishnan, V.: Linear Matrix Inequalities in System and Control Theory. SIAM, Philadelphia (1994)
37. Wu, M., He, Y., She, J.H.: Stability Analysis and Robust Control of Time-Delay Systems. Springer, New York (2010). https://doi.org/10.1007/978-3-642-03037-6

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

المنارة للاستشارات

www.manaraa.c

Reproduced with permission of copyright owner. Further reproduction prohibited without permission.

www.manaraa.c